

High frequency base balance methodologies for tall buildings with torsional and coupled resonant modes

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ABSTRACT: The paper discusses mode shape corrections and reviews processing methodologies for the determination of the overall wind loading and response of tall buildings using the high-frequency base balance technique. It is concluded that mode shape correction factors currently used for twist modes, are conservative. The effect of cross-correlations between base moments is found to be significant when calculating the response for coupled modes.

KEYWORDS: building, dynamic-response, high-frequency base balance, tall, wind loads

1 INTRODUCTION

The high-frequency base balance (HFBB) technique is now more than twenty years old [1], and has become the standard wind-tunnel method by which overall wind loads and responses such as accelerations, displacements and velocities are determined for tall buildings at the design stage. Essentially the mean (time-averaged) and quasi-static background base bending moments and torques are determined by direct measurement, but the resonant dynamic components are computed from the recorded time histories or spectral densities of the base moments. The simplicity of the wind-tunnel models, the rapidity at which tests can be carried out and the ease by which changes in basic dynamic properties such a frequency and damping can be incorporated greatly outweigh the disadvantage of the neglect of aeroelastic effects, generally regarded as negligible for the majority of habitable tall buildings.

However, although this is a well established technique, there are apparently still significant differences in the methods in use for dealing with :

- mode shape corrections for both sway and torsional, or twisting, dynamic modes
- coupled modes involving simultaneous sway and twist motions
- the redistribution of the final base moments as effective static wind forces over the height of the structure

This paper discusses the first two aspects of HFBB methodologies.

2 MODE SHAPE CORRECTIONS FOR SWAY AND TWIST MODES

2.1 Sway modes

In the following, it will be assumed that mode shapes in sway can be fitted by a power function of the form :

$$\mu(z) = \left(\frac{z}{h}\right)^\beta \tag{1}$$

A number of authors have considered the theoretical corrections required to correct the spectra of linearly weighted base bending moments to those for generalized forces. Holmes [2] derived the following correction factor assuming low correlation between the fluctuating sectional forces at any pair of height levels on the building. The spectral density of fluctuating sectional forces were assumed to be invariant with height.

$$S_{F_x}(n) = \left(\frac{1}{h}\right)^2 \left(\frac{3}{1+2\beta}\right) S_{M_y}(n) \tag{2}$$

Holmes [2] also derived the corresponding limit for full correlation of the fluctuating sectional forces, and proposed the following as an intermediate correction factor between the low and high correlation limits.

$$S_{F_x}(n) = \left(\frac{1}{h}\right)^2 \left(\frac{4}{1+3\beta}\right) S_{M_y}(n) \tag{3}$$

Boggs and Peterka [3] only considered the full correlation limit, but assumed that the fluctuating forces varied with height as a power law. Vickery *et al.* [4] made some direct measurements of the mode shape correction, using time histories of sectional forces based on pressure measurements. Xu and Kwok [5] modified the low and high limits of Holmes [2] to account for other variations of spectral density with height.

Table 1 summarizes the theoretical correction factors for various values of the mode shape exponent β , and of the exponent, γ , used to describe the variation of spectral density of fluctuating sectional forces with height in the form:

$$S_f(n, z) = S_f(n)_{\max} \cdot (z/h)^{2\gamma} \tag{4}$$

Table 1. Correction factors for spectral densities of generalized forces in sway modes

Mode- shape exponent β	Low correlation			High correlation			Recommended	Measured* Vickery <i>et al</i> [4]
	$\frac{3+2\gamma}{1+2\gamma+2\beta}$			$\left(\frac{2+\gamma}{1+\gamma+\beta}\right)^2$				
	$\gamma=0$	$\gamma=0.25$	$\gamma=0.5$	$\gamma=0$	$\gamma=0.25$	$\gamma=0.5$	$\frac{4}{1+3\beta}$	
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	-
1.25	0.86	0.88	0.89	0.79	0.81	0.83	0.84	0.86
1.5	0.75	0.78	0.80	0.64	0.67	0.69	0.73	0.71
2.0	0.60	0.64	0.67	0.44	0.48	0.51	0.57	0.61

The measurements of Vickery *et al* [4] were averages over the reduced frequency range of interest for the resonant response of tall buildings in urban exposures. For the mode shape exponents in the table, which cover the typical range for tall buildings, the measured factors

generally fall in between the high and low correlation limits, as expected. Note that assuming high correlation is unconservative for $\beta > 1.0$.

For practical purposes, a simple function, that does not require knowledge of the variation of spectral density with height, is desirable. This is provided by Equation (3), i.e. the function $4/(1+3\beta)$, suggested by Holmes [2]. As shown in Table 1, this fits the experimental data of Vickery *et al* [4] well. Another alternative would be the slightly more conservative low correlation limit for $\gamma = 0$, Equation (2), which also matches the experimental data well.

To determine the acceleration in this mode, the spectral density of the generalized force is factored by the transfer function for generalized acceleration. This involves the square of the generalized mass in the denominator. This is the mass per unit height each height level multiplied by the square of the mode shape, and integrated over the height of the structure. Then the effective correction factor for mean square acceleration (to correct from the assumption of a linear mode shape, $\beta = 0$) is:

$$h_a^2 = \left(\frac{4}{1+3\beta} \right) \left(\frac{1+2\beta}{3} \right)^2 \quad (5)$$

To determine the mean square resonant base moment, the moment due to the inertial forces arising from mass times acceleration at each height level is calculated. The resulting mode shape correction factor to the mean square resonant base bending moment is then :

$$h_M^2 = \left(\frac{4}{1+3\beta} \right) \left(\frac{1+2\beta}{2+\beta} \right)^2 \quad (6)$$

For a value of β of 1.5, the resultant correction factor for the mean square resonant base moment is 0.95. As the resonant component is usually about one half the total peak base bending moment the total error in neglecting mode shape corrections is typically only about 1-2%. Corrections to base bending moments are commonly ignored.

2.2 Twist modes

The HFFB measures a base torque uniformly weighted from the local torques per unit height over the full height of the building model. Since the mode shapes for the lowest twist mode of vibration of typical tall buildings increase monotonically with height from zero at the base, to a maximum value at or near the top of the building, large corrections are required when converting the measured fluctuating base torque to the fluctuating generalized force in the lowest twist mode of vibration. This contrasts with the situation with the lowest sway modes, for which the corrections to the measured base bending moments are small, and often neglected.

As for the sway modes, it will be assumed that mode shape can be fitted by a power function of the form:

$$\mu_t(z) = \left(\frac{z}{h} \right)^{\beta_t} \quad (7)$$

where β_t is the mode shape exponent for the twist mode shape described by a power law.

As for the sway modes, a power law variation of the spectral density of fluctuating sectional torque with height can be assumed :

$$S_t(n, z) = S_t(n)_{\max} \cdot (z/h)^{2\gamma} \tag{8}$$

Based on a uniform distribution ($\gamma=0$), and low correlation, the following correction factor, is obtained :

$$S_{Ft}(n) = \left(\frac{1}{1+2\beta_t} \right) S_{Mz}(n) \tag{9}$$

Alternative corrections, derived by Boggs and Peterka [3], allowed for variation of the fluctuating torque with height, but assumed full correlation of these fluctuating sectional torques over height separations. The theoretical correction factors obtained with assumptions of both high and low correlation, are shown in Table 2, together with the analysis of Tallin and Ellingwood [6], based on measurements of Reinhold [7].

Table 2. Correction factors for spectral densities of generalized forces in twist modes

β_t	Low correlation			High correlation			Recommended $\frac{1}{1+2\beta_t}$	Measured Tallin & Ellingwood [6]
	$\frac{1+2\gamma}{1+2\gamma+2\beta_t}$			$\left(\frac{1+\gamma}{1+\gamma+\beta_t} \right)^2$				
	$\gamma=0$	$\gamma=0.25$	$\gamma=0.5$	$\gamma=0$	$\gamma=0.25$	$\gamma=0.5$		
0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	-
0.75	0.40	0.50	0.57	0.33	0.39	0.44	0.40	-
1.0	0.33	0.43	0.50	0.25	0.31	0.36	0.33	0.32
1.25	0.29	0.38	0.44	0.20	0.25	0.30	0.29	-
1.5	0.25	0.33	0.40	0.16	0.21	0.25	0.25	0.26

Table 2 shows that a correction factor, based on low correlation and uniform fluctuating torques with height (Equation (9)), fits the data analysed by Tallin and Ellingwood well. This is not surprising as the correlations between fluctuating torques at various heights were computed by Tallin and Ellingwood, and were found to be very low.

At several wind-tunnel laboratories, a simple factor of 0.7 is used to convert the fluctuating measured base torque, $M_z(t)$, to the fluctuating generalized force in the twist mode, irrespective of the mode shape. This corresponds to a factor on the spectral density of $(0.7)^2 = 0.49$. As shown in Table 2, this is considerably greater than that obtained by Tallin and Ellingwood [6] from Reinhold's data, and appears to be over-conservative.

The correction factors for angular acceleration and resonant base torque are obtained in a similar way to the sway modes, giving Equations (10) and (11) respectively.

$$h_{at}^2 = \left(\frac{1}{1+2\beta_t} \right) \left(\frac{1+2\beta_t}{1} \right)^2 \tag{10}$$

$$\mathbf{h}_T^2 = \left(\frac{1}{1+2\beta_t} \right) \left(\frac{1+2\beta_t}{1+\beta_t} \right)^2 \quad (11)$$

3 COUPLED MODES OF VIBRATION

3.1 Processing methodologies

Many modern tall buildings have dynamic modes that involve simultaneous sway and twist motions. This often results from differences between the centre of mass and centre of stiffness (shear centre) of the cross-section. Several methods are available to process the outputs of the HFFB to make reasonable predictions of the resonant contributions from the coupled modes of tall buildings. These are summarized in the following.

Method 1. In this method, the spectra (i.e. auto spectral densities) of the three output signals proportional to M_x , M_y and M_z , are determined. Then the spectra of the generalized forces for the lower modes (usually three in number) are determined by linear weighting and summing of the resulting spectral densities. The relevant equations for three modes are as follows.

$$\begin{aligned} S_{F1}(n) &= \mathbf{h}_{1x}^2 (1/h^2) S_{My}(n) + \mathbf{h}_{1y}^2 (1/h^2) S_{Mx}(n) + \mathbf{h}_{1\theta}^2 S_{Mz}(n) \\ S_{F2}(n) &= \mathbf{h}_{2x}^2 (1/h^2) S_{My}(n) + \mathbf{h}_{2y}^2 (1/h^2) S_{Mx}(n) + \mathbf{h}_{2\theta}^2 S_{Mz}(n) \\ S_{F3}(n) &= \mathbf{h}_{3x}^2 (1/h^2) S_{My}(n) + \mathbf{h}_{3y}^2 (1/h^2) S_{Mx}(n) + \mathbf{h}_{3\theta}^2 S_{Mz}(n) \end{aligned} \quad (12)$$

Weighting terms like \mathbf{h}_{1x} , \mathbf{h}_{1y} etc. allow for the contribution of each component of base moment in the generalized force, as well as the mode shape corrections as discussed earlier.

Method 2. Errors in neglecting any correlation between the three measured moments in Method 1, are avoided by directly forming the time histories of the generalized forces for each mode, by weighting the time histories of the measured base moments, and then calculating the spectral densities from the new time series.

$$\begin{aligned} F_1(t) &= \mathbf{h}_{1x} (1/h) M_y(t) + \mathbf{h}_{1y} (1/h) M_x(t) + \mathbf{h}_{1\theta} M_z(t) \\ F_2(t) &= \mathbf{h}_{2x} (1/h) M_y(t) + \mathbf{h}_{2y} (1/h) M_x(t) + \mathbf{h}_{2\theta} M_z(t) \\ F_3(t) &= \mathbf{h}_{3x} (1/h) M_y(t) + \mathbf{h}_{3y} (1/h) M_x(t) + \mathbf{h}_{3\theta} M_z(t) \end{aligned} \quad (13)$$

Method 3. The correlations between the measured base moments, M_x , M_y and M_θ , can be incorporated in Method (1) by including the additional cross-spectral density terms in equations for the spectral densities of the generalized forces for the coupled modes. This approach was described by Irwin and Xie [8]. For example, the equation for the generalized force in Mode 1 can be written.

$$\begin{aligned} S_{F1}(n) &= \mathbf{h}_{1x}^2 (1/h^2) S_{My}(n) + \mathbf{h}_{1y}^2 (1/h^2) S_{Mx}(n) + \mathbf{h}_{1\theta}^2 S_{M\theta}(n) \\ &+ 2\mathbf{h}_{1x}\mathbf{h}_{1y} (1/h^2) S_{MyMx}(n) + 2\mathbf{h}_{1x}\mathbf{h}_{1\theta} (1/h) S_{MyM\theta}(n) + 2\mathbf{h}_{1y}\mathbf{h}_{1\theta} (1/h) S_{MxM\theta}(n) \end{aligned} \quad (14)$$

This method should give similar results to Method 2.

Method 4. The Yip and Flay [9] approach is a sophisticated frequency domain method designed to account for the shortcomings of the previous methods – namely neglect of the cross-spectral terms in Method 1, and also the reliance on empirical or theoretical mode shape corrections in all three methods. The cross-spectral densities (both real and imaginary components) are represented by low-order polynomial expressions in two space variables with

unknown frequency-dependent coefficients. The full set of auto-spectra and cross spectra from a 5-component base balance test, are used to determine the unknown coefficients. With this information, the unknown spectra of the generalized forces of the coupled modes can be determined. This method has apparently not been used in commercial wind-tunnel practice, and will not be discussed further in this paper.

3.2 Test case

As an example to illustrate the differences in results obtained from Methods 1 to 3, the results of processing HFBB data from a tall building with significant coupling in two modes of vibration are presented. The proposed building was approximately 200 metres in height with the cross section shown in Figure 1. Due to an eccentricity in the lift core, the building has significant coupling between sway in the x-direction and twist in both Modes 1 and 3. Wind-tunnel tests were performed on a tall building model (scale 1:250) placed in Windtech's boundary-layer wind tunnel with a blockage-tolerant test section. The tower section of the model was attached to a high-frequency base balance. The axis convention adopted for the tests is shown in Figure 1. A sample rate of 512 samples per second was used to sample the three base bending moments, with a sample time of 64 seconds. The full-scale building had frequencies in the first three modes between 0.15Hz and 0.20Hz. Time histories and spectral densities of generalized forces for three modes were calculated, using the mode shape corrections for sway and twist described in Sections 2.1 and 2.2, respectively. The resonant response contributions to the total base moments, and the accelerations at the top of the building, were calculated using standard procedures, based on the white noise approximation.

Figure 2 shows the variation of spectral density of the generalized force for Mode 1 at the natural frequency of that mode, as a function of wind direction. Methods 2 and 3 give similar predictions as expected – any differences are due to statistical sampling, or numerical errors in the calculations. However, values from Method 1 are generally greater than those from Methods 2 and 3. This can be attributed to negative correlations between the moments M_y and M_z . This contributes to the largest cross-spectral term in Equation (14), and hence reduces the predicted generalized force from that given by Method 1, which ignores the cross-spectral terms.

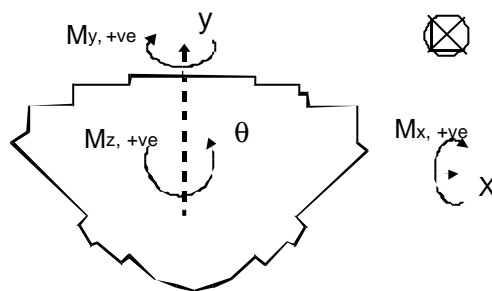


Figure 1. Building cross section and axis convention

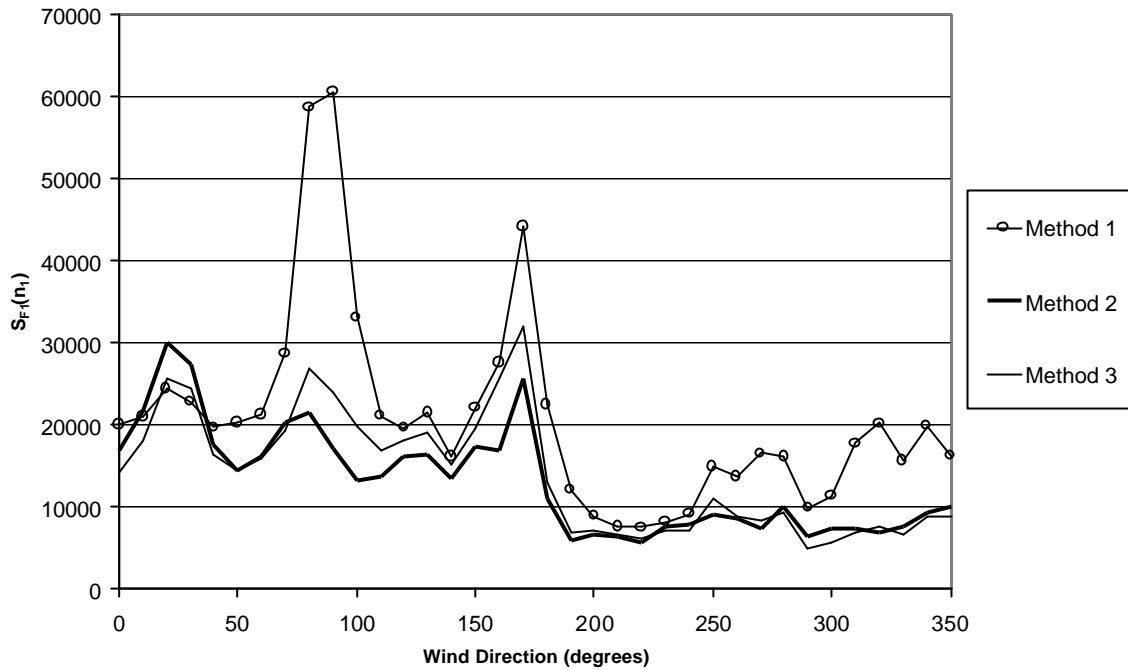


Figure 2. Comparison of the spectral density of generalized force (at frequency, n_1) for Mode 1 by three different methods

Table 3 gives the results of some final peak response predictions for the building. Coefficients of the base moments are defined in Equation (15). h is the building height, b is the maximum breadth, and \bar{U}_h is the mean wind speed at the top of the building.

$$C_{M_x} = \frac{M_x}{\frac{1}{2} \rho \bar{U}_h^2 b h^2} \quad C_{M_y} = \frac{M_y}{\frac{1}{2} \rho \bar{U}_h^2 b h^2} \quad C_{M_z} = \frac{M_z}{\frac{1}{2} \rho \bar{U}_h^2 b^2 h} \quad (15)$$

The maximum coefficient of the base bending moment, M_x , which is not affected by the coupling, is similar by all three Methods. However, Method 1 gives significant overestimates for M_y and M_z due to neglect of the cross-coupling terms in calculating the resonant contributions, as discussed previously. The resultant acceleration is also slightly overestimated by Method 1. However, neglect of any coupling (i.e. assuming that Modes 1, 2 and 3 are pure x-sway, y-sway and twist, respectively), results in even greater overestimates for M_y and M_z , but the resultant acceleration, in this case, is quite accurate.

Table 3. Comparison of response predictions for a building with two coupled modes

	5yr. Std. Devn. Acceleration (mg)	Coefft. of peak Base bending moment M_x	Coefft. of peak Base bending moment M_y	Coefft. of peak Base torque M_z
Method 1	3.0	1.30	0.64	0.40
Method 2	2.8	1.33	0.58	0.31
Method 3	2.7	1.31	0.56	0.32
Uncoupled	2.7	1.33	0.70	0.41

4 CONCLUSIONS

Mode shape corrections for the high-frequency base balance for both sway and twist modes have been reviewed. It is concluded that mode shape correction factors currently used for twist modes, are conservative, and a new function, based on the low correlation limit, is proposed.

Four methods of processing the recorded base moments to deal with resonant modes that have significant components of both sway and twist, have been discussed. The effect of cross-correlations between base moments was found to be significant when calculating the response for coupled modes. In this case, this resulted in overestimation of the resonant contributions to two of the base moments, when those terms were neglected (Method 1). This may not be a general conclusion for all buildings, however.

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